

Boclair Academy



Numeracy Booklet



RESPECT

HONESTY

FAIRNESS

AMBITION

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Estimation and Rounding

Third Level	Fourth Level
<ul style="list-style-type: none">I can round a number using an appropriate degree of accuracy, having taken into account the context of the problem	<ul style="list-style-type: none">Having investigated the practical impact of inaccuracy and error, I can use my knowledge of tolerance when choosing the required degree of accuracy to make real-life calculations

Rounding Whole and Decimal Numbers

Rounding a number makes it less accurate but often easier to use. When rounding, we need to consider the digit to the immediate right of the place value we want to round.

If the digit to the right is:

- 0, 1, 2, 3 or 4 then the digit we are rounding stays the same
- 5, 6, 7, 8 or 9 then the digit we are rounding increases by one

Examples:

1. Round 6829 to the nearest 1000.

Consider place value: Th H T U

6 8 2 9 → 7000



As the digit to the right of the thousands column is an 8, the 6 increases to a 7.

Similarly, to the nearest 100, $68\text{2}9 \rightarrow 6800$
to the nearest 10, $682\text{9} \rightarrow 6830$

We can also round decimals to a specified number of decimal places (d.p.) using this rule.

Examples:

2. Round 4.852 to
 - a) 1 decimal place.

We want one digit after the decimal point $4.8\text{5}2 \rightarrow 4.9$

- b) 2 decimal places.

We want two digits after the decimal point $4.85\text{2} \rightarrow 4.85$

3. Round 43.98 to 1 decimal place.

43.98 → 44.0



The 9 in the tenths column rounds up to 1 unit. The zero remains after the decimal point to show that the answer is rounded to 1 decimal place.

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Rounding to Significant Figures

Significant figures (sig. figs.) is another way of rounding which allows us to estimate the size of a number. The more significant figures we round to, the more accurate the number is.

To write a number to a specified number of significant figures we:

- Start at the first non-zero number and count along the specified number of digits
- Consider the next digit to the right and use the rounding rule outlined on the previous page

Examples:

1. A rugby match had an attendance of 47629. Round this figure to 1 significant figure.

To 1 sig. fig.: 47629 → 50000

first significant figure tells us how to round the first significant figure

Similarly, to 2 sig. figs.: 47629 → 48000

to 3 sig. figs.: 47629 → 47600

to 4 sig. figs.: 47629 → 47630

2. Round 0.20396 to 1 significant figure.

To 1 sig. fig.: 0.20396 → 0.2

first significant figure

Significant figures start from the first non-zero digit, so the zero in the units column here doesn't count

Similarly, to 2 sig. figs.: 0.20396 → 0.20

to 3 sig. figs.: 0.20396 → 0.204

to 4 sig. figs.: 0.20396 → 0.2040

We do not replace the numbers to the right of the decimal point with a zero as this would indicate they are significant.

Tolerance

Tolerance defines the range of values in which a measurement is acceptable.

In manufacturing, designs are drawn up to an exact size. But the process of cutting or moulding real materials means that some of the pieces will be a tiny amount larger than intended and some will be a tiny amount smaller than intended.

For example, a machine requires a part which is 45mm long. However, the part will still fit if it is 0.2mm too big or too small. Mathematically, this can be written as $45 \pm 0.2\text{mm}$. We read this as 45mm plus 0.2mm or 45mm minus 0.2mm. This means that the acceptable range of sizes for the part are from 44.8mm to 45.2mm.

Examples:

1. The tolerance for part of a curtain rail is $78.4 \pm 0.3\text{cm}$.
What are the largest and smallest acceptable sizes of the part?

$$\begin{aligned}\text{Largest size} &= 78.4 + 0.3 \\ &= 78.7\text{cm}\end{aligned}$$

$$\begin{aligned}\text{Smallest size} &= 78.4 - 0.3 \\ &= 78.1\text{cm}\end{aligned}$$

How can I practice **estimation and rounding** outside of school?

- Estimate how long it will take to drive/walk/cycle to a chosen location from your house.
- By rounding the cost of each item to the nearest pound, estimate how much this basket of shopping will cost?
- The website said 63467 tickets have been sold, how many is this to 1/2/3 sig. figs.?

Number and Number Processes

Third Level	Fourth Level
<ul style="list-style-type: none"> I can use a variety of methods to solve number problems in familiar contexts, clearly communicating my processes and solutions I can continue to recall number facts quickly and use them accurately when making calculations I can use my understanding of numbers less than zero to solve simple problems in context 	<ul style="list-style-type: none"> Having recognised similarities between new problems and problems I have solved before, I can carry out the necessary calculations to solve problems set in unfamiliar contexts I have investigated how introducing brackets to an expression can change the emphasis and can demonstrate my understanding by using the correct order of operations when carrying out calculations

Addition

Written Method

When adding numbers, ensure that they are lined up correctly according to their place value. Start at the right hand side, write down the units and carry tens to the next column on the left.

Examples:

1. Calculate $2715 + 386$.

$$\begin{array}{r}
 2715 \\
 + 386 \\
 \hline
 3101 \\
 \hline
 111
 \end{array}$$

Mental Strategies

There are a range of different strategies that can be applied to use addition mentally, a small sample of these are below.

Examples:

2. Calculate $28 + 64$

a) Add tens, add units and then add together

$$20 + 60 = 80 \quad 8 + 4 = 12 \quad 80 + 12 = 92$$

b) Split the second number in to units and tens and add in two steps

$$28 + 60 = 88 \quad 88 + 4 = 92$$

Subtraction

Written Method

When subtracting numbers, we write the larger number above the smaller number and ensure that they are lined up correctly according to their place value. We then start subtracting from the right column.

If the top digit is smaller than the bottom digit then we need to “borrow” from the next column to the left and add 10 to the top digit.

Examples:

1. Calculate $271 - 38$

$$\begin{array}{r} 271 \\ - 38 \\ \hline 233 \end{array}$$

2. Calculate $400 - 74$

$$\begin{array}{r} 400 \\ - 74 \\ \hline 326 \end{array}$$

Mental Strategies

Examples:

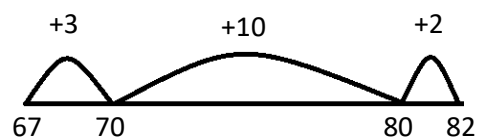
3. Calculate $82 - 67$

a) Decompose the number to be subtracted

$$67 = 60 + 7 \quad \text{so subtract 60 then subtract 7}$$

$$82 - 60 = 22 \quad 22 - 7 = 15$$

b) “Count On”



$$3 + 10 + 2 = 15$$

Multiplication

Written Method

When multiplying numbers, start at the right hand side, write down the units and carry tens to the next column on the left.

Examples:

1. Calculate 39×6 .

$$\begin{array}{r} 39 \\ \times 6 \\ \hline 234 \\ \hline \end{array}$$

2. John buys 8 packets of fruit pastilles. Each packet contains 12 pastilles. How many pastilles does John have in total?

$$\begin{array}{r} 12 \\ \times 8 \\ \hline 96 \\ \hline \end{array}$$

Mental Strategies

Examples:

3. Calculate 27×8

- a) Decompose any number larger than one digit

$27 = 20 + 7$ so multiply 8 by 20 then 8 by 7 and add answers together

$$20 \times 8 = 160 \quad 7 \times 8 = 56$$

$$\begin{aligned} 27 \times 8 &= 160 + 56 \\ &= 216 \end{aligned}$$

- b) Round to the nearest 10

$30 \times 8 = 240$ but 30 is three lots of 8 too many so subtract $3 \times 8 = 24$

$$240 - 24 = 216$$

Division

Written Method

When dividing numbers write the number to be divided inside the “bus stop” and the divisor outside. Start dividing from left to right, writing the answer above the line, moving any remainder on to the next digit.

Examples:

1. Calculate $696 \div 3$.

$$\begin{array}{r} 232 \\ 3 \overline{) 696} \end{array}$$

2. Calculate $120 \div 5$

$$\begin{array}{r} 024 \\ 5 \overline{) 120} \end{array}$$

Multiplication and Division by a Multiple of 10

When multiplying or dividing by a multiple of 10 (e.g. 40, 600, 3000, etc.) we carry out the calculation in two steps. First multiply/divide by the single digit and then multiply/divide by either 10, 100, 1000, etc.

Examples:

1. Calculate 53×30 .

$$\begin{array}{l} 53 \times 3 = 159 \quad \text{and} \quad 159 \times 10 = 1590 \\ \nearrow \quad \quad \quad \nearrow \\ \text{Multiply by 3} \quad \quad \text{Multiply by 10} \end{array}$$

2. Calculate 7.2×200 .

$$7.2 \times 2 = 14.4 \quad \text{and} \quad 14.4 \times 100 = 1440$$

3. Calculate $720 \div 60$.

$$\begin{array}{l} 720 \div 6 = 120 \quad \text{and} \quad 120 \div 10 = 12 \\ \nearrow \quad \quad \quad \nearrow \\ \text{Divide by 6} \quad \quad \text{Divide by 10} \end{array}$$

Long Multiplication

For long multiplication we multiply by the units and then by the tens before adding the resulting answers. This process continues in the same way when multiplying by numbers larger than two digits.

Examples:

1. Calculate 74×26

$$\begin{array}{r} 74 \\ \times 26 \\ \hline 444 \quad \leftarrow 74 \times 6 \\ + 1480 \quad \leftarrow 74 \times 20 \\ \hline 1924 \end{array}$$

2. Calculate 328×75

$$\begin{array}{r} 328 \\ \times 75 \\ \hline 1640 \quad \leftarrow 328 \times 5 \\ + 22960 \quad \leftarrow 328 \times 70 \\ \hline 24600 \\ \hline 11 \end{array}$$

Integers

The set of numbers known as integers comprises positive and negative whole numbers and the number zero. Integers can be represented on a number line.



Integers are used in a number of real life situations including temperature, scoring in sport, bank balances, profit and loss, elevators, etc.

Adding and Subtracting Integers

If you add a negative number, you get the same result as subtracting a positive number.

If you subtract a negative number, you get the same result as adding a positive number.

Examples:

1. Calculate $4 + (-3)$

$$\begin{aligned} 4 + (-3) &= 4 - 3 \\ &= 1 \end{aligned}$$

2. Calculate $3 + (-7)$

$$\begin{aligned} 3 + (-7) &= 3 - 7 \\ &= -4 \end{aligned}$$

3. Calculate $2 - (-3)$

$$\begin{aligned} 2 - (-3) &= 2 + 3 \\ &= 5 \end{aligned}$$

4. Calculate $(-9) - (-2)$

$$\begin{aligned} (-9) - (-2) &= (-9) + 2 \\ &= -7 \end{aligned}$$

Multiplying and Dividing Integers

To multiply or divide two integers, multiply the numbers first, then decide on the sign of your answer by using the following rules:

- If the sign of the numbers are the same, the answer will be positive.
- If the sign of the numbers are different, the answer will be negative.

Examples:

1. Calculate $4 \times (-3)$

$$4 \times (-3) = -12$$

2. Calculate $(-7) \times (-7)$

$$(-7) \times (-7) = 49$$

3. Calculate $(-15) \div 3$

$$(-15) \div 3 = -5$$

4. Calculate $(-9) \div (-3)$

$$(-9) \div (-3) = 3$$

Order of Operations

Calculations which involve more than one operation (addition, subtraction, multiplication or division) have to be completed in a specific order.

The mnemonic “BODMAS” helps us to remember the correct order to complete operations.

Brackets

Order

Division

Multiplication

Addition

Subtraction

× and ÷ have the same priority

+ and – have the same priority

Operations with the same priority are completed from left to right.

Examples:

1. Calculate $31 - 18 \div 3$

$$\begin{aligned} 31 - 18 \div 3 & \quad \text{division first} \\ & = 31 - 6 \quad \text{then subtraction} \\ & = 25 \end{aligned}$$

2. Calculate $4 \times (3 + 2)$

$$\begin{aligned} 4 \times (3 + 2) & \quad \text{bracket first} \\ & = 4 \times 5 \quad \text{then multiplication} \\ & = 20 \end{aligned}$$

3. Calculate $10 - (13 + 2) \div 5$

$$\begin{aligned} 10 - (13 + 2) \div 5 & \quad \text{bracket first} \\ & = 10 - 15 \div 5 \quad \text{then division} \\ & = 10 - 3 \quad \text{then subtraction} \\ & = 7 \end{aligned}$$

4. Calculate $10 + 2^2 - 6$

$$\begin{aligned} 10 + 2^2 - 6 & \quad \text{order (powers) first} \\ & = 10 + 4 - 6 \quad \text{same priority, left to right} \\ & = 14 - 6 \quad \text{then subtraction} \\ & = 8 \end{aligned}$$

How can I practice **number and number processes** outside of school?

- There are 45 people attending a party and each person will eat approximately 3 sausage rolls. How many sausage rolls do I need to buy?
- Calculate the cost of one can of juice when buying a multipack from the supermarket.
- Calculate differences in temperature from one day to another.

Fractions, Decimal Fractions and Percentages

Third Level	Fourth Level
<ul style="list-style-type: none"> I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations I can show how quantities that are related can be increased or decreased proportionally and apply this to solve problems in everyday contexts 	<ul style="list-style-type: none"> I can choose the most appropriate form of fractions, decimal fractions and percentages to use when making calculations mentally, in written form or using technology, then use my solutions to make comparisons, decisions and choices Using proportion, I can calculate the change in one quantity caused by a change in a related quantity and solve real-life problems

Fraction of a Quantity

To find a fraction of a quantity you divide the quantity by the denominator and then multiply this answer by the numerator (if the numerator is one then we do not need to complete this step as multiplying by one leaves the term unchanged).

Examples:

1. Calculate $\frac{1}{2}$ of 26

$$26 \div 2 = 13$$

2. Calculate $\frac{1}{5}$ of 125

$$125 \div 5 = 25$$

3. Calculate $\frac{3}{4}$ of 80

$$80 \div 4 = 20$$

$$20 \times 3 = 60$$

4. Calculate $\frac{2}{5}$ of 85

$$85 \div 5 = 17$$

$$17 \times 2 = 34$$

Adding and Subtracting Fractions

When adding and subtracting fractions, the denominators of the fractions must be the same. If the denominators are different then we find the lowest common multiple of the denominators and use equivalent fractions. Once the denominators are the same, we simply add or subtract the numerators and then simplify if possible.

Examples:

1. Calculate $\frac{1}{2} + \frac{3}{4}$

$$\begin{aligned} & \frac{1}{2} + \frac{3}{4} \\ &= \frac{2}{4} + \frac{3}{4} \\ &= \frac{5}{4} \text{ or } 1\frac{1}{4} \end{aligned}$$

The lowest common multiple of 2 and 4 is 4, so we need to find an equivalent fraction to $\frac{1}{2}$ with 4 on the denominator

2. Calculate $\frac{4}{7} - \frac{2}{5}$

$$\begin{aligned} & \frac{4}{7} - \frac{2}{5} \\ &= \frac{20}{35} - \frac{14}{35} \\ &= \frac{6}{35} \end{aligned}$$

The lowest common multiple of 7 and 5 is 35, so we need to find equivalent fractions to $\frac{4}{7}$ and $\frac{2}{5}$ with 35 on the denominator

3. Calculate $\frac{2}{3} - \frac{5}{12}$

$$\begin{aligned} & \frac{2}{3} - \frac{5}{12} \\ &= \frac{8}{12} - \frac{5}{12} \\ &= \frac{3}{12} \\ &= \frac{1}{4} \end{aligned}$$

Remember to simplify a fraction as far as possible

4. Calculate $2\frac{3}{4} + 3\frac{1}{2}$

$$\begin{aligned} & 2\frac{3}{4} + 3\frac{1}{2} \\ &= \frac{11}{4} + \frac{7}{2} \\ &= \frac{11}{4} + \frac{14}{4} \\ &= \frac{25}{4} \text{ or } 6\frac{1}{4} \end{aligned}$$

Convert mixed numbers to improper fractions first and then add as normal

5. Calculate $2\frac{2}{5} - 1\frac{1}{6}$

$$\begin{aligned} & 2\frac{2}{5} - 1\frac{1}{6} \\ &= \frac{12}{5} - \frac{7}{6} \\ &= \frac{72}{30} - \frac{35}{30} \\ &= \frac{37}{30} \text{ or } 1\frac{7}{30} \end{aligned}$$

Multiplying Fractions

When multiplying fractions we multiply the numerators together, multiply the denominators together and then simplify if possible.

Examples:

1. Calculate $\frac{2}{7} \times \frac{4}{9}$

$$\begin{aligned} & \frac{2}{7} \times \frac{4}{9} \\ &= \frac{8}{63} \end{aligned}$$

2. Calculate $\frac{3}{10} \times \frac{4}{7}$

$$\begin{aligned} & \frac{3}{10} \times \frac{4}{7} \\ &= \frac{12}{70} \\ &= \frac{6}{35} \end{aligned}$$

3. Calculate $2\frac{2}{5} \times 4\frac{2}{3}$

$$\begin{aligned} & 2\frac{2}{5} \times 4\frac{2}{3} \\ &= \frac{12}{5} \times \frac{14}{3} \\ &= \frac{168}{15} \\ &= \frac{56}{5} \text{ or } 11\frac{1}{5} \end{aligned}$$

Dividing Fractions

When dividing by a fraction it is easier to obtain an answer by multiplying by the reciprocal of the dividing fraction. To do this we can remember **copy, change, flip**.

We **copy** the first fraction, **change** the divide sign to multiply, and **flip** the last fraction upside down (to obtain the reciprocal). We then multiply the fractions as demonstrated earlier.

Examples:

1. Calculate $\frac{1}{2} \div \frac{1}{4}$

$$\frac{1}{2} \div \frac{1}{4}$$

$$= \frac{1}{2} \times \frac{4}{1}$$

$$= \frac{4}{2}$$

$$= 2$$

2. Calculate $\frac{2}{5} \div \frac{3}{5}$

$$\frac{2}{5} \div \frac{3}{5}$$

$$= \frac{2}{5} \times \frac{5}{3}$$

$$= \frac{10}{15}$$

$$= \frac{2}{3}$$

3. Calculate $3\frac{3}{5} \div 2\frac{5}{8}$

$$3\frac{3}{5} \div 2\frac{5}{8}$$

$$= \frac{18}{5} \div \frac{21}{8}$$

$$= \frac{18}{5} \times \frac{8}{21}$$

$$= \frac{144}{105}$$

$$= \frac{48}{35} \text{ or } 1\frac{13}{35}$$

Decimals

The value of a digit in a decimal (or decimal fraction), is determined by place value. This enables us to convert between decimals and fractions.

Examples:

1. Convert 0.614 to a fraction.

Units . Tenths Hundredths Thousandths

0 . 6 1 4

The 6 can be written as $\frac{6}{10}$; the 1 as $\frac{1}{100}$ and the 4 as $\frac{4}{1000}$.

$$\begin{aligned} & \frac{6}{10} + \frac{1}{100} + \frac{4}{1000} \\ = & \frac{600}{1000} + \frac{10}{1000} + \frac{4}{1000} \\ = & \frac{614}{1000} \\ = & \frac{307}{500} \end{aligned}$$

2. Convert the following decimals to fractions:

a) 0.6

$$\begin{aligned} &= \frac{6}{10} \\ &= \frac{3}{5} \end{aligned}$$

b) 0.62

$$\begin{aligned} &= \frac{62}{100} \\ &= \frac{31}{50} \end{aligned}$$

c) 0.128

$$\begin{aligned} &= \frac{128}{1000} \\ &= \frac{16}{125} \end{aligned}$$

When completing basic calculations with decimals, be careful that you align the decimal points.

Examples:

1. Calculate $2.7 + 3.83$

$$\begin{array}{r} 2.70 \\ + 3.83 \\ \hline 6.53 \\ \hline \end{array}$$

Fill any spaces with a zero

2. Calculate 3.19×4

$$\begin{array}{r} 3.19 \\ \times 4 \\ \hline 12.76 \\ \hline \end{array}$$

3. Calculate $10.5 \div 2$

$$\begin{array}{r} 05.25 \\ 2 \overline{) 10.50} \\ \hline \end{array}$$

If you have a remainder at the end of a calculation, add a zero to the end of the decimal and continue

Percentages

Percent means one part in every hundred. So, for example, 23% is 23 out of 100 and can be written as $\frac{23}{100}$. As a decimal, 23% can be written as $23 \div 100 = 0.23$.

To convert a decimal to a percentage multiply the decimal by 100. For example, $0.58 \times 100 = 58\%$.

To convert a fraction to a percentage, first find the related decimal by dividing the numerator by the denominator and then by multiply by 100. For example, $\frac{2}{5} = 0.4 = 40\%$.

It is expected that the equivalent fractions and decimals are known for the following commonly used percentages:

Percentages	Fraction	Decimal
1%	$\frac{1}{100}$	0.01
10%	$\frac{1}{10}$	0.1
20%	$\frac{1}{5}$	0.2
25%	$\frac{1}{4}$	0.25
50%	$\frac{1}{2}$	0.5
75%	$\frac{3}{4}$	0.75

Calculating Percentages

There are several ways of calculating percentages, including using equivalent fractions, decimals or even directly on a calculator.

Examples:

1. Calculate 25% of £320

$$25\% = \frac{1}{4}$$

$$\frac{1}{4} \text{ of } £320$$

$$= 320 \div 4$$

$$= £80$$

2. Calculate 30% of 4000m

$$10\% = 4000 \div 10$$

$$= 400\text{m}$$

$$30\% = 400 \times 3$$

$$= 1200\text{m}$$

3. Calculate 8% of 300kg

$$1\% = 300 \div 100$$

$$= 3\text{kg}$$

$$8\% = 3 \times 8$$

$$= 24\text{kg}$$

4. Calculate 47% of £400

$$47\% = 40\% + 7\%$$

$$10\% = 400 \div 10$$

$$= £40$$

$$40\% = 40 \times 4$$

$$= £160$$

$$47\% = 160 + 28$$

$$= £188$$

$$1\% = 400 \div 100$$

$$= £4$$

$$7\% = 4 \times 7$$

$$= £28$$

5. Using a calculator, calculate 42% of 920g

$$42\% = \frac{42}{100}$$

$$\frac{42}{100} \times 920$$

$$= 386.4\text{g}$$

6. In the Scottish Premiership, 84 penalties were taken last season. 75% of these penalties were scored. Calculate how many penalties were scored.

75% of 84

$$= \frac{3}{4} \text{ of } 84$$

$$84 \div 4 = 21$$

$$21 \times 3 = 63$$

63 penalties were scored

7. An employee is given a 4% wage rise. If his current weekly wage is £480 per week, calculate his new wage after the wage rise.

Pay rise is 4% of £480

$$\frac{4}{100} \times 480$$

$$= £19.20$$

$$\text{New wage} = 480 + 19.20$$

$$= £499.20$$

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Writing Ratios

When we write ratios we write the items and quantities in the same order.

For example, the instructions to make diluting juice say, mix ten parts water to one part concentrate.

The ratio of water to concentrate is 10 : 1 (read as “10 to 1”).

Whereas, the ratio of concentrate to water is 1 : 10 (read as “1 to 10”).

Simplifying Ratios

To write a ratio in its simplest form, we divide each number by the highest common factor of the terms in the ratio, similar to when simplifying fractions.

Examples:

1. Simplify 20 : 8

20 : 8 The highest common factor of 20 and 8 is 4, so
5 : 2 divide both numbers by 4

2. The colour turquoise is made by mixing 20 parts of blue paint with 10 parts of green paint. Write the ratio of blue to green paint in its simplest form.

Blue : Green
20 : 10
2 : 1

Ratio Calculations

We can solve problems using ratios, if we are given one quantity and a known ratio.

Examples:

1. The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit : Nut
 $\times 5 \left(\begin{array}{c} 3 : 2 \\ 15 : 10 \end{array} \right) \times 5$

2. A company selling soft drinks creates their drink using the ratio 5 parts water to 2 parts syrup. How much water is required for an 8 litre bag of syrup?

Water : Syrup
 $\times 4 \left(\begin{array}{c} 5 : 2 \\ 20 : 8 \end{array} \right) \times 4$ 20 litres of water is required.

Sharing in a Given Ratio

It is possible to split a total quantity in a given ratio by calculating the total number of “shares”, working out the value of each “share” and then finding the value of each section of the ratio.

Examples:

1. Laura and Paula earn money by selling lemonade. As Laura did more of the preparation work, they have decided to share the profit of £40 in the ratio 3 : 2. How much money will Laura and Paula receive?

Total shares	$= 3 + 2$	Add the numbers in the ratio together to get total “shares”
	$= 5$	

Value of 1 share	$= 40 \div 5$	Divide the total amount by the number of shares
	$= 8$	

Laura gets	$= 3 \times 8$	Multiply the value of 1 share by the number of shares from each part of the ratio
	$= £24$	
Paula gets	$= 2 \times 8$	
	$= £16$	

2. A lottery win of £500000 is shared amongst Louise, James and Mohammed in the ratio 3 : 5 : 2. How much does each person receive?

Total shares	$= 3 + 5 + 2$
	$= 10$

Value of 1 share	$= 500000 \div 10$
	$= 50000$

Louise gets	$= 3 \times 50000$
	$= £150000$

James gets	$= 5 \times 50000$
	$= £250000$

Mohammed gets	$= 2 \times 50000$
	$= £100000$

Direct Proportion

Two quantities are said to be in direct proportion if, as one quantity increases, the other quantity increases in the same ratio.

Examples:

1. The cost of 5 cakes is £3.50. Calculate the cost of 7 cakes.

$$5 \text{ cakes} = £3.50$$

$$1 \text{ cake} = 3.50 \div 5$$

$$= £0.70$$

$$7 \text{ cakes} = 0.70 \times 7$$

$$= £4.90$$

Find the cost of 1 first, and
then find the cost of 7

Indirect Proportion

Two quantities are said to be in indirect proportion if, as one quantity increases, the other quantity decreases in the same ratio.

Examples:

1. It takes 3 painters 6 hours to complete a job. How long would it have taken 4 painters?

$$3 \text{ painters} = 6 \text{ hours}$$

$$1 \text{ painter} = 6 \times 3$$

$$= 18 \text{ hours}$$

$$4 \text{ painters} = 18 \div 4$$

$$= 4.5 \text{ hours}$$

Find the time for 1 painter first,
and then find the time for 4

How can I practice **fractions, decimal fractions and percentages** outside of school?

- A cake contains 1640 calories. How many calories are in $\frac{1}{8}$ of the cake?
- A shop is offering 30% off all clothes. How much will a jacket that costs £108 cost in the sale?
- If 5 cookies cost £3.85, calculate the cost of 8 cookies.

Money

Third Level	Fourth Level
<ul style="list-style-type: none"> When considering how to spend my money, I can source, compare and contrast different contracts and services, discuss their advantages and disadvantages, and explain which offer best value to me I can budget effectively, making use of technology and other methods, to manage money and plan for future expenses 	<ul style="list-style-type: none"> I can discuss and illustrate the facts I need to consider when determining what I can afford, in order to manage credit and debt and lead a responsible lifestyle I can source information on earnings and deductions and use it when making calculations to determine net income I can research, compare and contrast a range of personal finance products and, after making calculation, explain my preferred choices

Currency Exchange

The currency used in the United Kingdom is Pounds Sterling (£s). Other currencies that you may be familiar with are the Euro (€) and the US Dollar (\$).

We use exchange rates to convert between currencies. The exchange rate between countries changes all the time, depending on the financial situations in different countries.

To change Pounds Sterling to another currency, we multiply the money we have by the exchange rate.

To change foreign currency back to Pounds sterling, we divide the money we have by the exchange rate.

Examples:

1. Scott goes on holiday to Rome and takes £800 spending money with him. Using the exchange rate below, how many Euros will Scott buy?

Exchange Rate: £1 = 1.15€

$$\begin{aligned}\text{Euros} &= 800 \times 1.15 \\ &= 920\text{€}\end{aligned}$$

2. Innaya returns from a school trip to New York with \$70. Using the exchange rate below, calculate how many pounds Innaya will receive when she converts her money back.

Exchange Rate: £1 = \$1.32

$$\begin{aligned}\text{Pounds} &= 70 \div 1.32 \\ &= 53.0303... \\ &= \text{£}53.03\end{aligned}$$

Calculating 'Best Buys'

Often when shopping we want to calculate which size of product gives us the best value for money. To do this we are aiming to calculate the cost per gram, bottle, 100ml, etc. Doing this then allows us to compare the two products and select the one which gives the best value for money.

Examples:

1. A pack of 8 cans of cola cost £2.48.
A pack of 14 cans of cola cost £4.06.
Which pack offers the best value for money?

8 cans = £2.48	14 cans = £4.06
1 can = $2.48 \div 8$	1 can = $4.06 \div 14$
= £0.31	= £0.29

The 14 pack of cans offers the best value for money.

2. Which packet of mince offers the best value?
 - 500 grams at £5.30
 - 400 grams at £4.20
 - 250 grams at £2.70

500 grams = £5.30	400 grams = £4.20	250 grams = £2.70
100 grams = $5.30 \div 5$	100 grams = $4.20 \div 4$	100 grams = $2.70 \div 2.5$
= £1.06	= £1.05	= £1.08

The 400 grams packet of mince offers the best value.

How can I practice money outside of school?

- Calculate how many euros you will receive for the spending money you have saved.
- Calculate which item offers the best value for money when purchasing goods.
- Compare mobile phone contracts over a 24 month period and identify ways of saving money over the contract period.

Time

Third Level	Fourth Level
<ul style="list-style-type: none">Using simple time periods, I can work out how long a journey will take, the speed travelled at or distance covered, using my knowledge of the link between time, speed and distance	<ul style="list-style-type: none">I can research, compare and contrast aspects of time and time management as they impact on me

12 and 24 Hour Clock

Time is measured using either the 12 or 24 hour clock. When using the 12 hour clock, a.m. represents from midnight to noon and p.m. from noon to midnight. Times written using the 24 hour clock require 4 digits ranging from 00:00 to 23:59. Midnight is expressed as 00:00 and 12 noon is written as 12:00.

Examples:

1.	<table><tr><th>12 hour</th><th>24 hour</th></tr><tr><td>2.45 a.m.</td><td>02:45</td></tr><tr><td>7.14 a.m.</td><td>07:14</td></tr><tr><td>Noon</td><td>12:00</td></tr><tr><td>5.29 p.m.</td><td>17:29</td></tr><tr><td>10.38 p.m.</td><td>22:38</td></tr><tr><td>Midnight</td><td>00:00</td></tr></table>	12 hour	24 hour	2.45 a.m.	02:45	7.14 a.m.	07:14	Noon	12:00	5.29 p.m.	17:29	10.38 p.m.	22:38	Midnight	00:00
12 hour	24 hour														
2.45 a.m.	02:45														
7.14 a.m.	07:14														
Noon	12:00														
5.29 p.m.	17:29														
10.38 p.m.	22:38														
Midnight	00:00														

Time Intervals

When calculating time intervals we do not use subtraction. We can use the adding on method as demonstrated below.

Examples:

1. How long is it from 12:40 to 16:15?

12:40 → 13:00 → 16:00 → 16:15
20 mins 3 hours 15 mins = 3 hours and 35 minutes

2. How long is it from 6.25 a.m. to 1.47 p.m.?

6:25 a.m. → 7.00 a.m. → 1.00 p.m. → 1:47 p.m.
35 mins 6 hours 47 mins = 6 hours and 82 minutes
= 7 hours and 22 minutes

Converting Minutes to Hours

To convert from minutes to decimal hours we divide the minutes by 60.

To convert from decimal hours to minutes we multiply the decimal hours by 60.

Examples:

1. Convert 36 minutes to hours.

$$36 \div 60 = 0.6$$

$$36 \text{ minutes} = 0.6 \text{ hours}$$

2. Convert 2 hours and 21 minutes to hours.

$$21 \div 60 = 0.35$$

$$2 \text{ hours and } 21 \text{ minutes} = 2.35 \text{ hours}$$

3. Convert 0.9 hours to minutes.

$$0.9 \times 60 = 54$$

$$0.9 \text{ hours} = 54 \text{ minutes}$$

4. Convert 3.15 hours to hours and minutes.

$$0.15 \times 60 = 9 \text{ minutes}$$

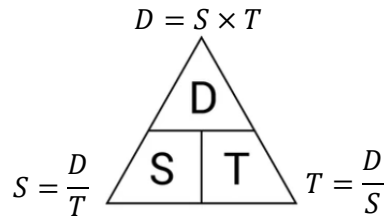
$$3.15 \text{ hours} = 3 \text{ hours and } 9 \text{ minutes}$$

Speed, Distance and Time

By applying our knowledge of time, we can carry out calculations involving speed and distance using the following formulae:

$$\text{Distance} = \text{Speed} \times \text{Time} \qquad \text{Speed} = \frac{\text{Distance}}{\text{Time}} \qquad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

The following triangle is often used to help remember these formulae:



Examples:

1. Calculate the average speed of a train which travelled 460km in 5 hours.

$$\begin{aligned} D &= 460\text{km} \\ S &= ? \\ T &= 5 \text{ hours} \end{aligned} \qquad \begin{aligned} S &= \frac{D}{T} \\ &= \frac{460}{5} \\ &= 92\text{km/h} \end{aligned}$$

2. A runner can run at a pace of 4 miles per hour. What distance does she cover if she runs at that pace for 2 hours and 30 minutes?

$$\begin{aligned} D &= ? \\ S &= 4\text{mph} \\ T &= 2 \text{ hours and } 30 \text{ minutes} \\ &= 2.5 \text{ hours} \end{aligned} \qquad \begin{aligned} D &= S \times T \\ &= 4 \times 2.5 \\ &= 10\text{miles} \end{aligned}$$

3. A bus travels 129.6 miles at an average speed of 36 miles per hour. How long does the journey take?

$$\begin{aligned} D &= 129.6 \text{ miles} \\ S &= 36\text{mph} \\ T &= ? \end{aligned} \qquad \begin{aligned} T &= \frac{D}{S} \\ &= \frac{129.6}{36} \\ &= 3.6 \text{ hours} \\ &= 3 \text{ hours and } 36 \text{ minutes} \end{aligned}$$

How can I practice **time** outside of school?

- The film starts at 5.40 p.m. and lasts for 2 hours and 37 minutes. When will it end?
- We need to arrive in Glasgow by 16:20. What train should we get to arrive on time?
- If I can cycle 12 miles per hour, how long will it take me to cycle 42 miles?

Measurement

Third Level	Fourth Level
<ul style="list-style-type: none"> I can solve practical problems by applying my knowledge of measure, choosing the appropriate units and degree of accuracy for the task, and using a formula to calculate area or volume when required 	<ul style="list-style-type: none"> I can apply my knowledge and understanding of measure to everyday problems and tasks and appreciate the practical importance of accuracy when making calculations

Units of Length, Volume and Weight

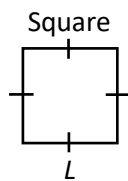
The following conversions are commonly used when undertaking tasks involving measurement:

Length	Volume	Weight
10mm = 1cm	1cm ³ = 1ml	1000g = 1kg
100cm = 1m	1000ml = 1L	
1000m = 1km	1000cm ³ = 1L	

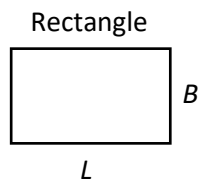
Area

Area is defined as the amount of space taken up by a 2D shape. 2D shapes are completely flat. When calculating areas we used squared units, for example, cm² or m².

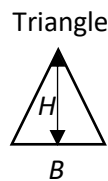
The formulae used for calculating the area of (some) simple 2D shapes are below:



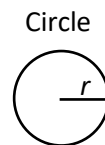
$$\text{Area} = L^2$$



$$\text{Area} = L \times B$$



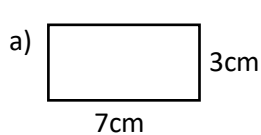
$$\text{Area} = \frac{1}{2} B \times H$$



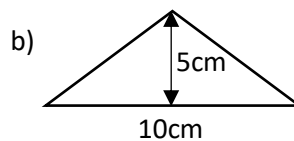
$$\text{Area} = \pi r^2$$

Examples:

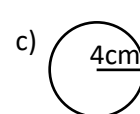
1. Calculate the area of the following shapes:



$$\begin{aligned} A &= L \times B \\ &= 7 \times 3 \\ &= 21\text{cm}^2 \end{aligned}$$



$$\begin{aligned} A &= \frac{1}{2} B \times H \\ &= \frac{1}{2} \times 10 \times 5 \\ &= 25\text{cm}^2 \end{aligned}$$

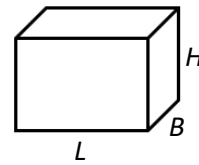


$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 4^2 \\ &= 50.2654... \\ &= 50.3\text{cm}^2 \end{aligned}$$

Volume

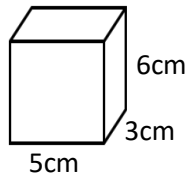
Volume is defined as the amount of space taken up by a 3D shape. When calculating volumes we used squared units, for example, cm^3 or m^3 . If the shape is filled with a liquid then we can use units such as millilitres and litres.

To calculate the volume of a cuboid we use the formula: $V = L \times B \times H$.



Examples:

1. Calculate the volume of the cuboid below.



$$\begin{aligned} V &= L \times B \times H \\ &= 5 \times 3 \times 6 \\ &= 90\text{cm}^3 \end{aligned}$$

How can I practice **measurement** outside of school?

- How many packs of flooring do we need to replace the flooring in the kitchen?
- Calculate the area of the wall to be painted and how many tins of paint are required.
- Weigh out ingredients for making a meal.

Data and Analysis

Third Level	Fourth Level
<ul style="list-style-type: none"> I can work collaboratively making appropriate use of technology, to source information presented in a range of ways, interpret what it conveys and discuss whether I believe the information to be robust, vague or misleading. 	<ul style="list-style-type: none"> I can evaluate and interpret raw and graphical data using a variety of methods, comment on relationships I observe within the data and communicate my findings to others.

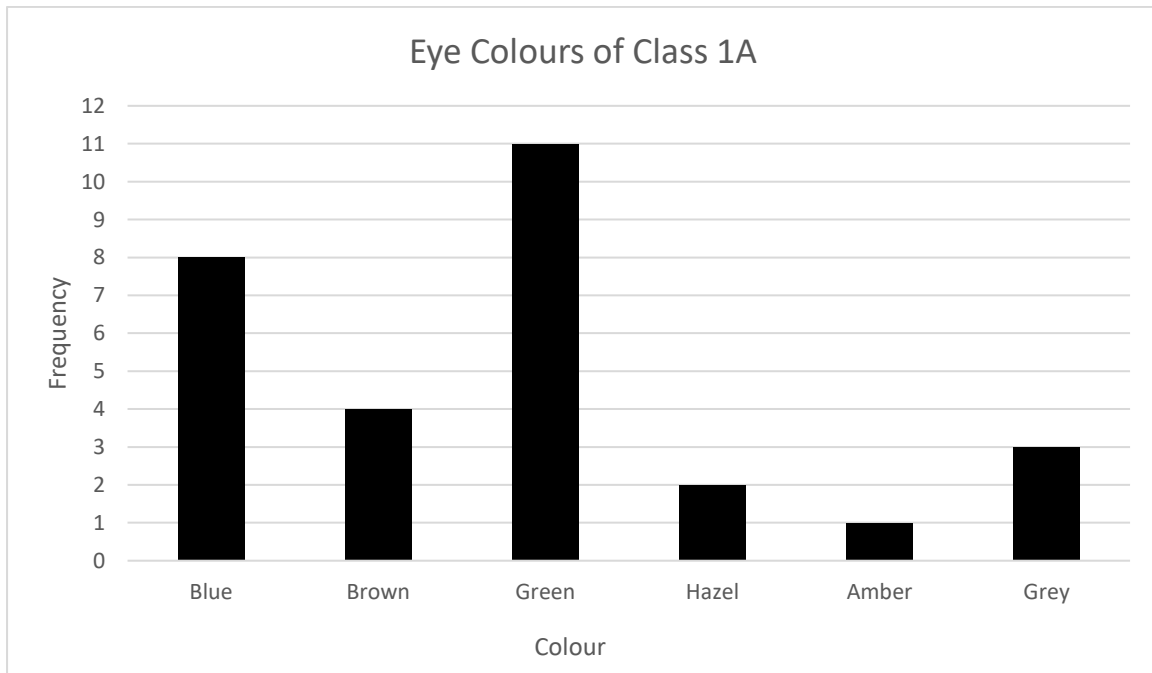
Frequency Tables

Frequency tables allow us to group data in order to interpret the data more easily. We use tally marks to help count the frequency.

Eye Colour	Tally	Frequency
Blue		8
Brown		4
Green		11
Hazel		2
Amber		1
Grey		3

Bar Graphs

Bar graphs are used to display discrete data (data that can be grouped). These graphs must have equal spaces between the bars and the bars must be the same width. Bar graphs should include a title and labels on both axes.



RESPECT

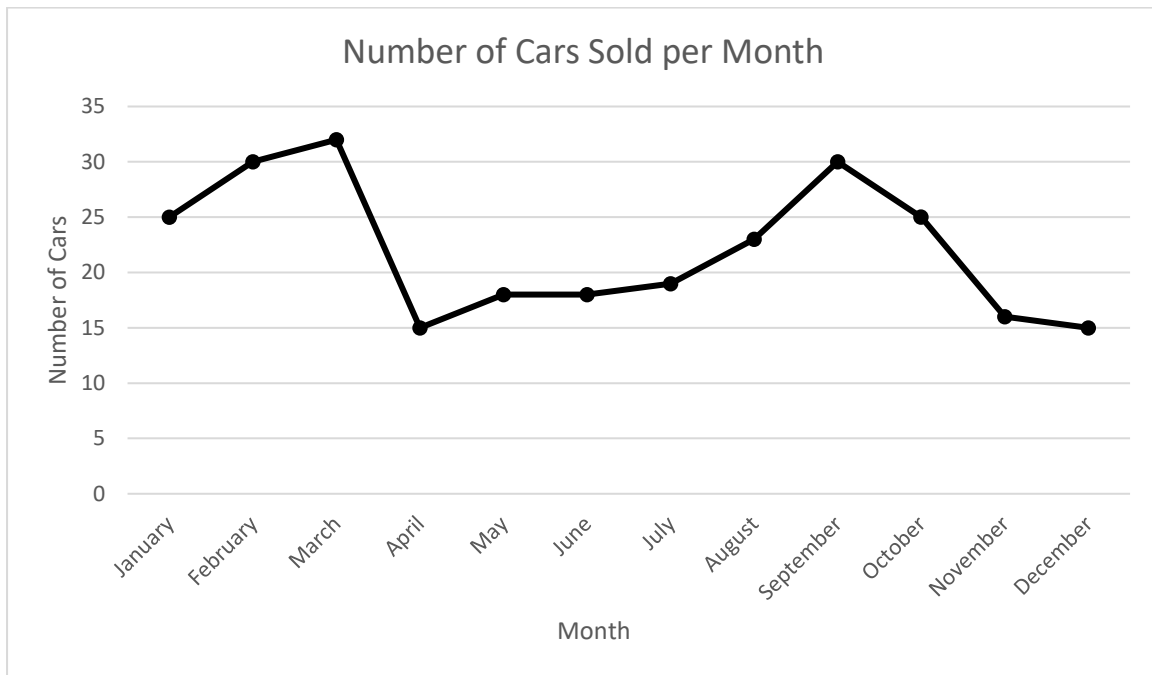
HONESTY

FAIRNESS

AMBITION

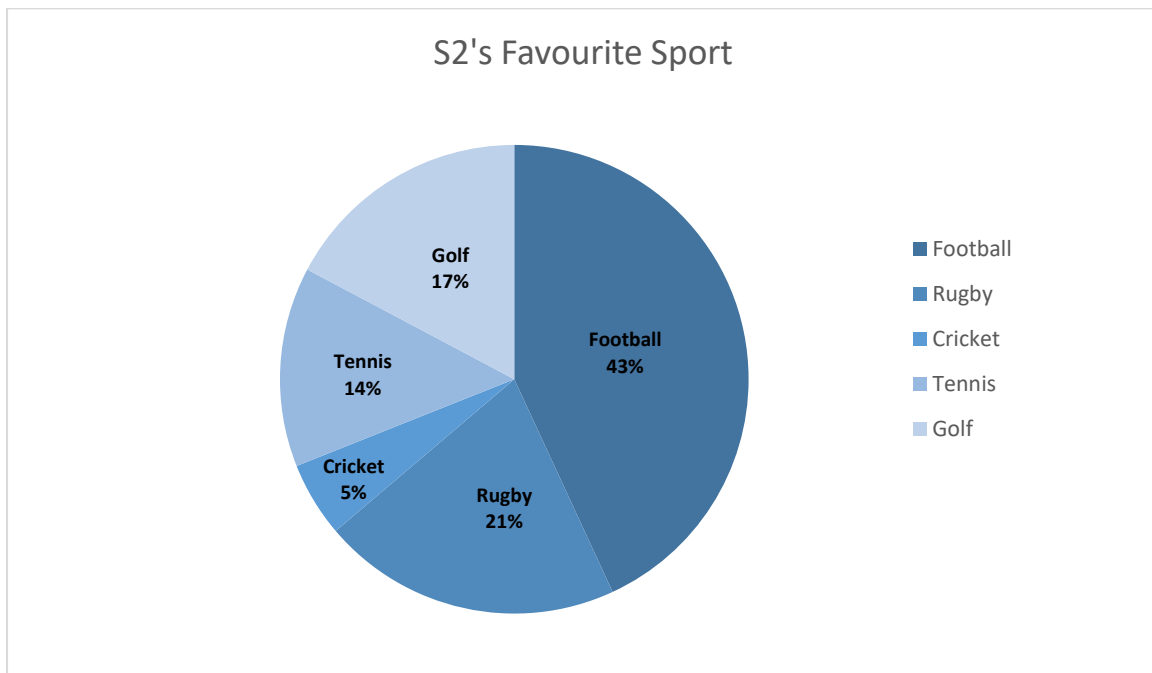
Line Graphs

Line graphs are used to display changes in data over a period of time. These graphs consist of a series of points that are joined up. Line graphs should include a title and labels on both axes.



Pie Charts

Pie charts are used to display how a whole set is divided into parts, showing the proportion of each part to the total. These graphs are circular, each sector must be labelled (and/or a key used) and the chart must have a title.



RESPECT

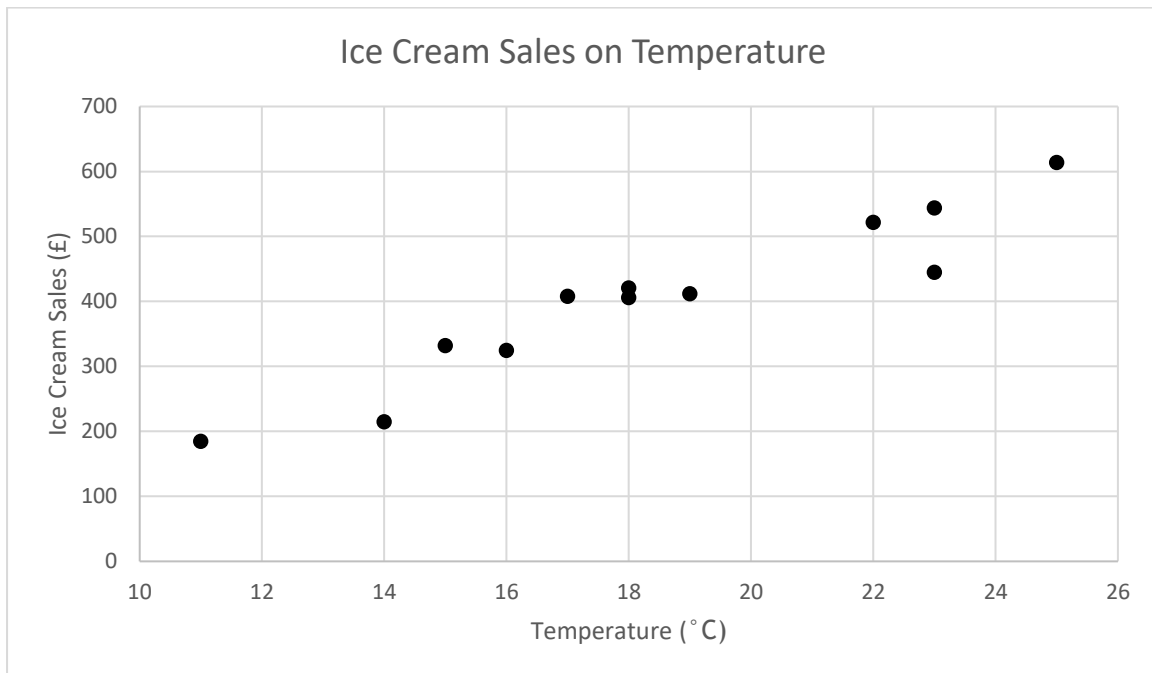
HONESTY

FAIRNESS

AMBITION

Scatter Graphs

Scatter graphs are used to display two sets of data, in order to see if a relationship exists between the two. Scatter graphs must include a title and labels on both axes.

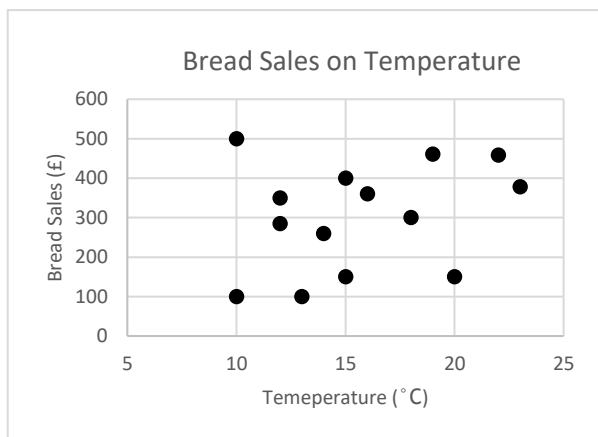
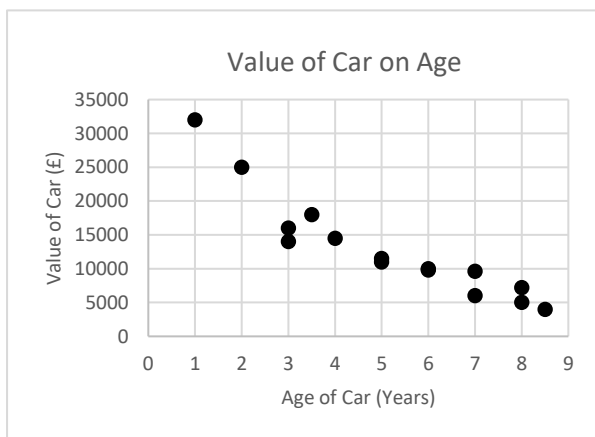


Correlation

We can describe the relationship between the two variables by looking for a correlation. The example above shows a **positive correlation** because, in general, as the temperature increases the ice cream sales also increase.

A **negative correlation**, shown on the left below, occurs if an increase in one variable leads to a decrease in the other. For example, the age of a car and the value of the car.

If the two variables do not appear to follow any pattern then we say there is **no correlation**, shown on the right below.



RESPECT

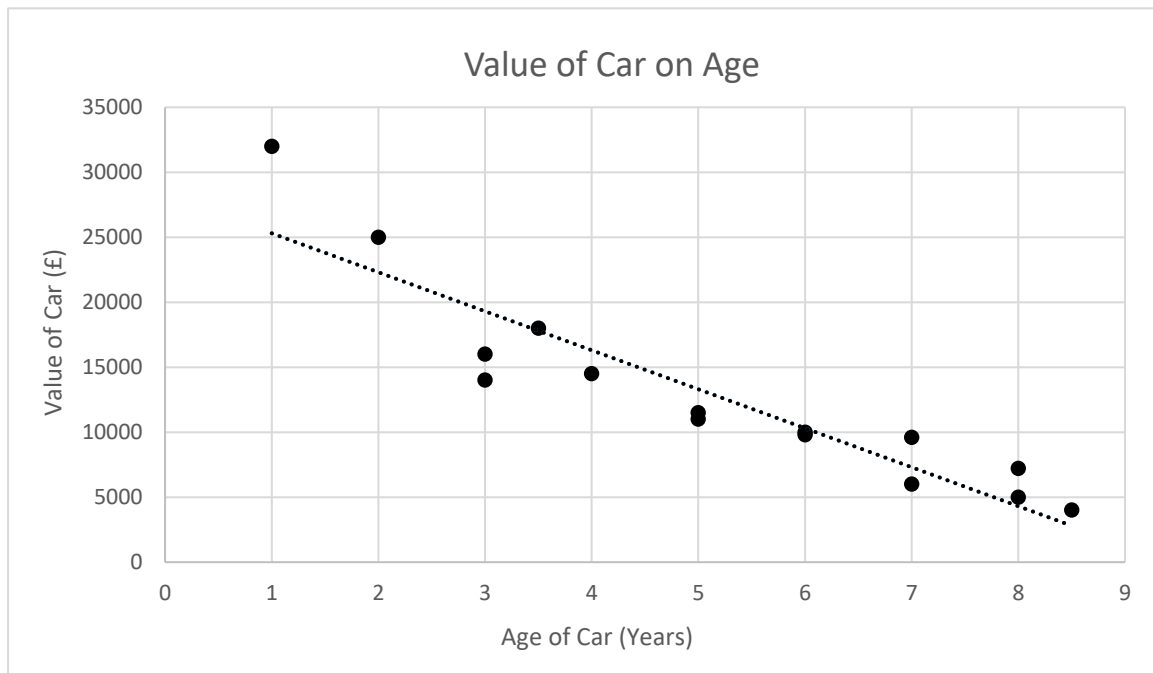
HONESTY

FAIRNESS

AMBITION

Line of Best Fit

A line of best fit is a straight line which is used to emphasise the relationship between two variables on a scatter graph. The aim is to get the line to go through as many points as possible, while getting roughly the same number of the remaining points on either side of the line. Once a line of best fit has been drawn, it can be used to estimate values.



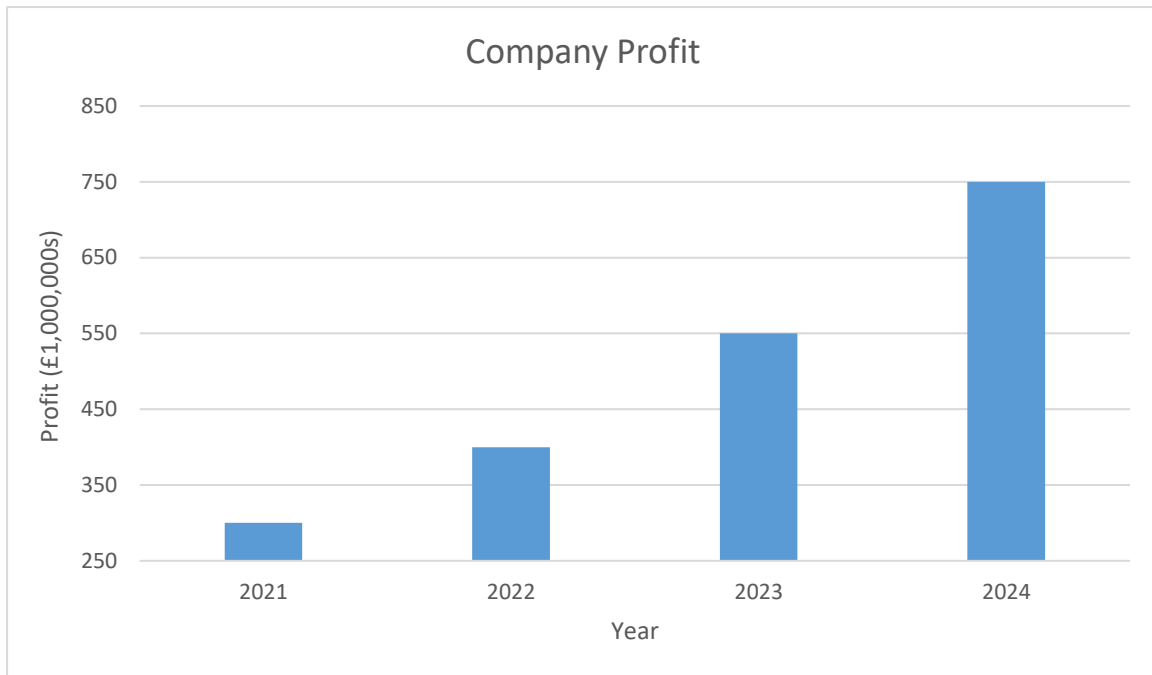
From our line of best fit, we can estimate that a car that is 5.5 years old would cost approximately £1200.

Misleading Data

Graphs are used to make information seem credible. However, graphs should be read with a critical eye as the graph may misrepresent or skew data to support certain narratives.

Common ways of doing this are; omitting the baseline, inconsistent scales on the y-axis, disregarding some data and using the wrong type of graph.

For example, by omitting the baseline below, the company profits in 2023 and 2024 appear exaggerated.



How can I practice **data and analysis** outside of school?

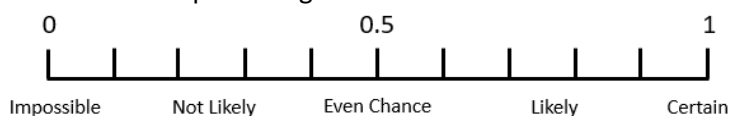
- Record the temperature of each day during the school holidays and then display the data in a statistical chart.
- Find three different articles on a newspaper site where statistical charts have been used. Check if the data is misleading in any way.
- To the nearest hour, record your screen time for a whole month and display the data in a frequency table.

Ideas of Chance and Uncertainty

Third Level	Fourth Level
<ul style="list-style-type: none"> I can find the probability of a simple event happening and explain why the consequences of the event, as well as its probability, should be considered when making choices. 	<ul style="list-style-type: none"> By applying my understanding of probability, I can determine how many times I expect an event to occur, and use this information to make predictions, risk assessment, informed choices and decisions.

Probability

Many events can't be predicted with total certainty. However, probability helps us to describe the likelihood (or chance) of the event happening. The probability of an event is how likely the event is to happen and is measured on a scale from 0 to 1. Probability is often written as a fraction, but can easily be converted to a decimal or percentage.



Calculating Probability

When calculating probability we want to know the fraction of the time something will happen. To calculate probability, we use the formula:

$$\text{Probability (event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Examples:

1. If I roll a 6-sided dice, what is the probability that it lands on:



- a) An even number

$$P(\text{even}) = \frac{\text{fav}}{\text{poss}}$$

$$P(\text{even}) = \frac{3}{6}$$

$$P(\text{even}) = \frac{1}{2}$$

favourable outcomes are the things we want, i.e. an even number (2, 4 and 6)

possible outcomes all the things we could get, i.e. all number on a 6-sided dice (1, 2, 3, 4, 5 and 6)

- b) The number 0

$$P(0) = \frac{\text{fav}}{\text{poss}}$$

$$P(0) = \frac{0}{6}$$

$$P(0) = 0$$

- c) A number greater than 4

$$P(> 4) = \frac{\text{fav}}{\text{poss}}$$

$$P(> 4) = \frac{2}{6}$$

$$P(> 4) = \frac{1}{3}$$

Comparing Probabilities

We often want to compare different probabilities in order to see which event is more likely to occur. To do this we have to change our fraction in to a decimal or percentage.

Examples:

- Two bags contain different numbers of coloured balls.
Bag 1 contains 4 red balls, 7 blue balls and 3 yellow balls.
Bag 2 contains 3 red balls, 2 blue balls and 5 yellow balls.
Which bag gives the best opportunity of choosing a red ball?

Bag 1

$$P(\text{red}) = \frac{\text{fav}}{\text{poss}}$$

$$P(\text{red}) = \frac{4}{14}$$

$$P(\text{red}) = 0.286$$

$$P(\text{red}) = 28.6\%$$

Bag 2

$$P(\text{red}) = \frac{\text{fav}}{\text{poss}}$$

$$P(\text{red}) = \frac{3}{10}$$

$$P(\text{red}) = 0.3$$

$$P(\text{red}) = 30\%$$

Bag 2 gives the best chance of choosing a red ball as $30\% > 28.6\%$.

How can I practice **ideas of chance and uncertainty** outside of school?

- Play card or dice games and use your knowledge of probabilities to make better decisions
- Play “Higher or Lower” with a deck of cards and use probabilities to work your way across the board
- Rank the likelihood of events happening on a probability scale