# Boclair Academy

# Mathematics Department

## Consistent Approaches to

## Learning and Teaching Methods

## Adding and Subtracting fractions

*Example:*

$$3\frac{3}{4} + 1\frac{2}{5}$$

$$= \frac{15}{4} + \frac{7}{5}$$

$$= \frac{75}{20} + \frac{28}{20}$$

$$= \frac{103}{20}$$

- Convert fractions into improper fractions
- Obtain a common denominator using LCM or by multiplying denominators (avoid smile & kiss)
- Multiply to get numerators as appropriate
- Add/subtract numerators
- Simplify as fully as possible

*Example:*

$$2\frac{4}{5} - \frac{7}{8}$$

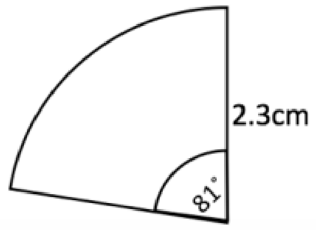
$$= \frac{14}{5} - \frac{7}{8}$$

$$= \frac{112}{40} - \frac{35}{40}$$

$$= \frac{77}{40}$$

**Arc Length and Sector Area**

Example:




Calculate the area of the sector.

$$\frac{\theta}{360} = \frac{A}{\pi r^2} = \frac{L}{\pi D}$$

- Write out the 3 ratios

$$\frac{\theta}{360} = \frac{A}{\pi r^2}$$

- Select the 2 ratios needed to answer the question

$$\frac{81}{360} = \frac{A}{\pi\,(2.3)^2}$$

- Substitute in what you know

$$A = \frac{81 \times \pi \times 2.3^2}{360}$$

- Rearrange formula

$$A = 3.7392\ldots$$

- Calculate the answer

$$A = 3.74\ cm^2$$

The same process would happen for calculating the length of an arc. Select the length and angle ratios.

**Standard Deviation**

The price, in pence, of petrol at 5 petrol stations are:

121, 119, 120, 117, 118

Calculate the mean and standard deviation.

$$\overline{x} = \frac{121\ +\ 119\ +\ 120\ +\ 117\ +\ 118}{5} = 119$$

- Calculate the mean

| $x$ | $x - \overline{x}$ | $(x - \overline{x})^2$ |
|---|---|---|
| 121 | 121 - 119 = 2 | 4 |
| 119 | 119 - 119 = 0 | 0 |
| 120 | 120 - 119 = 1 | 1 |
| 117 | 117 - 119 = -2 | 4 |
| 118 | 118 - 119 = -1 | 1 |
| | $\sum(x - \overline{x})^2$ = 10 | |

- Set up table with 3 columns as shown
- Column 1 is each piece of data; column 2 is each piece of data subtracting the mean; column 3 is column 2 squared.

- Add up all the digits in the final column

SD = $\sqrt{\frac{\Sigma(x - \underline{x})^2}{n-1}} = \sqrt{\frac{10}{4}}$

= 1.5811…

= 1.58

- Write out the formula
- The numerator is the total of the final column

- *n* is the number of data entries there are

- Make sure you square root the whole fraction

- Round answer to 2 decimal places

**Equation of a straight line**

Example:

Calculate the equation of the line AB where A is (-2, 5) and B is (4, -3).

- Calculate the gradient by finding the difference between in the vertical divided by the difference in the horizontal

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-3 - 5}{4 - (-2)}$$

- Simplify gradient as far as possible

$$m = \frac{-8}{6} = \frac{-4}{3}$$

- Substitute the gradient and one point into the general equation of a line

$$\boldsymbol{y - b = m(x - a)}$$

$$y - 5 = -\frac{4}{3}(x + 2)$$

- Rearrange equation into y = mx + c

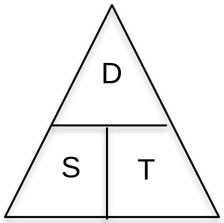
$$3y - 15 = -4x - 8$$

$$y = -\frac{4}{3}x - \frac{7}{3}$$

**Speed, distance, time**

Example:

A car travels for 2 and half hours and covers a distance of 80 miles.
Calculate the average speed of the car.




S = ?
D = 80
T = 2.5

S = D ÷ T

= 80 ÷ 2.5

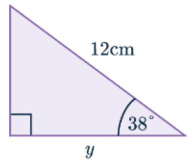
= 32 MPH

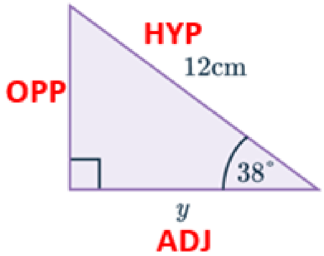
**The triangle helps you know what calculation to make. When the letters are next to each other, this means multiply. When the letters are on top of each other then you divide the top by the bottom.*

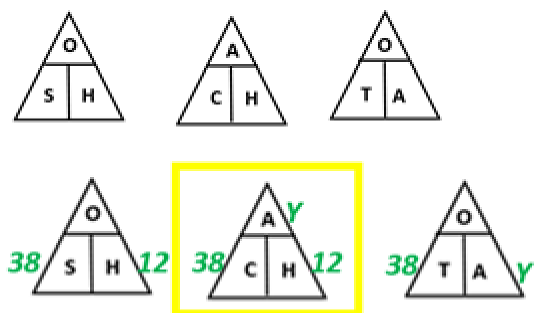
- Draw out the SDT triangle
- Write down S, D, T and identify what information you know.
- Cover up the letter of the thing you know
- Write out the correct calculation
- Substitute in the values
- Calculate correctly

## Right Angle Trigonometry - length of a side

Example: Calculate the length of y




$\text{adj} \quad = \cos x \times hyp$

$adj = \cos 38 \times 12$

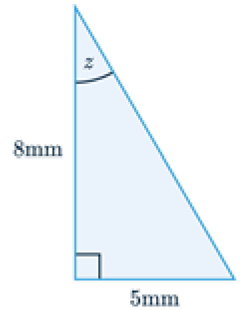
$= 11.46\ldots$

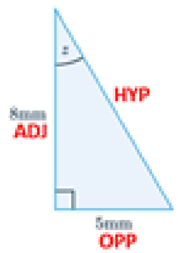
$y \quad = 11.5\text{cm}$

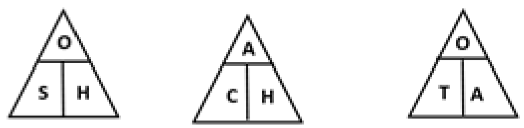
- Label the triangle with opposite, adjacent and hypotenuse
- Write out formula triangles
- Write the information from the triangle into the formula triangles
- Select rule with something written on each side
- Cover up what you want, write out rule and substitute in
- Carefully calculate using your calculator

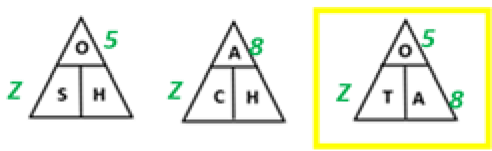
**Right Angle Trigonometry - size of an angle**

Calculate the size of angle z.




$tan(x) = \frac{opp}{adj}$

$tan(z) = \frac{5}{8}$

z = $tan^{-1}\frac{5}{8}$

z = 32º

- Label the triangle with opposite, adjacent and hypotenuse
- Write out formula triangles
- Write the information from the triangle into the formula triangles
- Select rule with something written on each side
- Write out formula and substitute in
- Use inverse [sin,cos or tan] to calculate the angle
- On your calculator, look for *shift* or *2nd F* then hit sin, cos or tan.

**Expanding Brackets**

Example:

Multiply out and simplify

(3x - 1)(x + 4)

$$3x(x + 4) - 1(x + 4)$$

$$= 3x^2 + 12x - x - 4$$

$$= 3x^2 + 11x - 4$$

- Split first bracket into single terms and multiply by the second bracket
- Multiply each term
- Simplify as appropriate (collect like terms)

Example:

Expand and simplify

$$(x + 5)(x^2 + 6x - 2)$$

$$(x(x^2 + 6x - 2) + 5(x^2 + 6x - 2)$$

$$x(x^2 + 6x - 2) + 5(x^2 + 6x - 2)$$

$$= x^3 + 6x^2 - 2x + 5x^2 + 30x - 10$$

$$= x^3 + 11x^2 + 28x - 10$$

## Factorising trinomials

Example:

Factorise $x^2 + 11x + 24$

| + | x |
|---|---|
| 11 | 24 |
| 8, 3 | |

$x^2 + 11x + 24$

- Find what numbers multiply to make the last term and adds to make the middle

$x^2 + 8x + 3x + 24$

- Rewrite the expression with the 2 numbers as coefficients of x

$x^2 + 8x + 3x + 24$

- Split down the middle and factorise each side for a common factor

$x(x + 8) + 3(x + 8)$

$\underline{(x + 8)(x + 3)}$

- Take one of the brackets as part of the final answer and then the terms in front of the brackets as the other.

Example:

Factorise $6x^2 + 13x + 6$

| + | x |
|---|---|
| 13 | 36 |
| 9, 4 | |

$6x^2 + 13x + 6$

- Multiply the first by last term when coefficient of $x^2 > 1$
- Find what numbers multiply to make this and adds to make the middle term

$6x^2 + 9x + 4x + 6$

- Rewrite the expression with the 2 numbers as coefficients of x

$6x^2 + 9x + 4x + 6$

- Split down the middle and factorise each side for a common factor

$3x(2x + 3) + 2(2x + 3)$

$\underline{(3x + 2)(2x + 3)}$

- Take one of the brackets as part of the final answer and then the terms in front of the brackets as the other.

## Reverse Percentages

Example:

The value of a house after an increase of 12% is £210,000.

Calculate what it originally cost.

100% + 12% = 112%

112% = 210,000

1% = 210000 ÷ 112

= 1875

100% = 1875 × 100

= £187,500

- Identify the percentage increase or decrease
- State what the percentage is equal to
- Calculate 1%
- Calculate 100%

**Compound Interest**

Example:

A car decreases in value by 3% each year. When James bought the car, it cost him £12,450. Calculate what it will be worth after 5 years.

100% - 3% = 97%

97% $\Rightarrow$ 0.97

$12450 \times 0.97^5$ = £10,691.238…

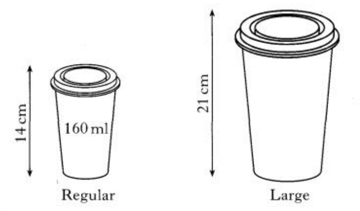
= £10, 691.24

- Calculate percentage increase or decrease from 100%
- Convert to a decimal
- Multiply the original by the multiplier and put this to the power of time frame.
- Calculate answer on calculator

# Similarity

Example:

**7.** Coffee is sold in regular cups and large cups.
The two cups are mathematically similar in shape.




The regular cup is 14 centimetres high and holds 160 millilitres.
The large cup is 21 centimetres high.
Calculate how many millilitres the large cup holds.

LSF = $\frac{21}{14}$

VSF = $(\frac{21}{14})^3$

New volume = $(\frac{21}{14})^3$x 160

= 540ml

LSF = Linear scale factor = $\frac{new}{original}$

ASF = Area scale factor = $(\frac{new}{original})^2$

VSF = Volume scale factor = $(\frac{new}{original})^3$

- Calculate the scale factor by dividing the measurement from the shape you need to know about divided by the original length
- For a length, multiply by the scale factor by the corresponding length
- For an area, square the scale factor and multiply by the corresponding area
- For a volume, cube the scale factor and multiply by the corresponding volume

## Completing the square

| | | |
|---|---|---|
| $x^2 + 6x + 5$ | *Side working* | |
| $(\mathrm{x} + 3)^2 - 3^2 + 5$ | $(x^2 + 6x + \ldots) + 5$ | • Recognise that $x^2 + 6x$ has come from a quadratic and need to put back into brackets |
| $(\mathrm{x} + 3)^2 - 9 + 5$ | *From* $(\mathrm{x} + 3)^2$ | • Put this into a bracket and subtract the square of the single number explaining where this has come from. |
| $\underline{(\mathrm{x} + 3)^2 - 4}$ | | • Simplify |

$2x^2 - 8x + 7$

$2[x^2 - 4x] + 7$

$2[(x - 2)^2 - 4] + 7$

$\underline{2(x - 2)^2 - 1}$

- For examples with $x^2$ coefficient >1 then take out the factor through the $x^2$ and x term using square brackets.

## Solving equations

Example: Solve 5x + 4 = 14

$$5x + 4 = 14$$
$$(-4) \qquad (-4)$$
$$\frac{5x}{5} = \frac{10}{5}$$
$$\underline{x = 2}$$

Example: Solve 12x - 20 = 15x - 38

$$12x - 20 = 15x - 38$$
$$(-12x) \qquad (-12x)$$
$$-20 = 3x - 38$$
$$(+38) \qquad (+38)$$
$$\frac{18}{3} = \frac{3x}{3}$$
$$\underline{x = 6}$$

- Use the balancing method - ‘*whatever you do to one side, you do to the other*’

- When dividing, put the whole of the numerator over the divisor to encourage students to divide everything by the same term

**Simultaneous Equations**

Example:

$4x - 2y = 4$ *(1)* (x3)

$5x + 3y = 16$ *(2)* (x2)

$12x - 6y = 12$

$10x + 6y = 32$

$22x = 44$

$x = 2$

Sub x = 2 into (1)

$4(2) - 2y = 4$

$8 - 2y = 4$

$-2y = -4$

$y = 2$

Example:

$2x + 4y = 24$ *(1)* (x -3)

$4x + 3y = 22$ *(2)* (x 4)

$-8x - 12y = -72$

$16x + 12y = 88$

$8x = -16$

$x = -2$

Sub x = -2 into (1)

$2(-2) + 4y = 24$

$-4 + 4y = 24$

$4y = 28$

$y = 7$

- Ensure x or y have the same number in both equations by multiplying the equation by a factor
- Eliminate one term by adding - terms you want to eliminate should have one negative, one positive in either equation.
- Solve for one term
- Substitute term into one equation
- Calculate other term
- For equations where all terms are positive, multiply through one equation by a negative, then add like terms.

**Solving trigonometric equations**

Example:
Solve $3\,sinx + 2 = 0$ $(0 \leq x \leq 360)$

- Begin to solve equation as normal

$3\,sinx + 2 = 0$
(-2) (-2)
$3sinx = -2$
(÷3) (÷3)
$sinx = -\frac{2}{3}$

The negative tells you if we are looking for the solution above or below the x

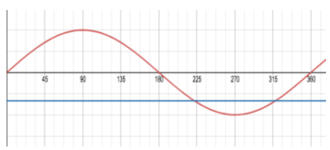
- To solve for x then you use inverse/shift of the trig ratio. This should only be taken of the positive value.

$(sin^{-1}(\frac{2}{3}))$
(41.8 RAA)

- Make a rough sketch of the graph and identify where on the graph the line meets it.
- Decide if you subtract/add to 180 or subtract from 360 depending on the graph.

$x = 180 + 41.8$
$= \underline{221.8^{o}}$

$x = 360 - 41.8$
$x = \underline{318.2^{o}}$



- The cast diagram will then give a shortcut for finding where the graph is positive.

As pupils become familiar with the graphs then the CAST diagram can be used.

| | |
|---|---|
| S | A |
| T | C |